
mud Documentation

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This is the documentation of **mud**.

CONTENTS**1.1 readme****1.1.1 MUD**

Analytical solutions and some associated utility functions for computing maximal updated density points for Data-Consistent Inversion.

1.1.2 Description

Maximal Updated Density Points are the values which maximize an updated density, analogous to how a MAP (Maximum A-Posteriori) point maximizes a posterior density from Bayesian inversion. Updated densities differ from posteriors in that they are the solution to a different problem which seeks to match the push-forward of the updated density to a specified observed distribution.

More about the differences here...

What does this package include?

1.2 License

The MIT License (MIT)

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1.3 Contributors

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1.4 mud

1.4.1 mud package

Submodules

mud.base module

class mud.base.BayesProblem(*X*, *y*, *domain=None*)

Bases: `object`

Sets up Bayesian Inverse Problem for parameter identification

```
>>> from mud.base import BayesProblem
>>> import numpy as np
>>> from scipy.stats import distributions as ds
>>> X = np.random.rand(100,1)
>>> num_obs = 50
>>> Y = np.repeat(X, num_obs, 1)
>>> y = np.ones(num_obs)*0.5 + np.random.randn(num_obs)*0.05
>>> B = BayesProblem(X, Y, np.array([[0,1], [0,1]]))
>>> B.set_likelihood(ds.norm(loc=y, scale=0.05))
>>> np.round(B.map_point()[0],1)
0.5
```

`estimate()`

`fit()`

`map_point()`

`set_likelihood(distribution, log=False)`

`set_prior(distribution=None)`

class mud.base.DensityProblem(*X*, *y*, *domain=None*)

Bases: `object`

Sets up Data-Consistent Inverse Problem for parameter identification

```
>>> from mud.base import DensityProblem
>>> from mud.funs import wme
>>> import numpy as np
>>> X = np.random.rand(100,1)
>>> num_obs = 50
>>> Y = np.repeat(X, num_obs, 1)
>>> y = np.ones(num_obs)*0.5 + np.random.randn(num_obs)*0.05
>>> W = wme(Y, y)
>>> B = DensityProblem(X, W, np.array([[0,1], [0,1]]))
>>> np.round(B.mud_point()[0],1)
0.5
```

```

estimate()
fit(**kwargs)
mud_point()
set_initial(distribution=None)
set_observed(distribution=<scipy.stats._distn_infrastructure.rv_frozen object>)
set_predicted(distribution=None, **kwargs)

```

mud.funs module

Python console script for *mud*, installed with *pip install .* or *python setup.py install*

```

mud.funs.check_args(A, b, y, mean, cov, data_cov)
mud.funs.iterate(A, b, y, initial_mean, initial_cov, data_cov=None, num_epochs=1, idx=None)
mud.funs.main(args)
    Main entry point allowing external calls

    Parameters args ([str]) – command line parameter list

mud.funs.makeRi(A, initial_cov)
mud.funs.map_problem(lam, qoi, qoi_true, domain, sd=0.05, num_obs=None, log=False)
    Wrapper around map problem, takes in raw qoi + synthetic data and instantiates solver object
mud.funs.map_sol(A, b, y=None, mean=None, cov=None, data_cov=None, w=1, return_pred=False)
mud.funs.mud_problem(lam, qoi, qoi_true, domain, sd=0.05, num_obs=None, split=None)
    Wrapper around mud problem, takes in raw qoi + synthetic data and performs WME transformation, instantiates
    solver object
mud.funs.mud_sol(A, b, y=None, mean=None, cov=None, data_cov=None, return_pred=False)
    For SWE problem, we are inverting N(0,1). This is the default value for data_cov.
mud.funs.mud_sol_alt(A, b, y=None, mean=None, cov=None, data_cov=None, return_pred=False)
    Doesn't use R directly, uses new equations. This presents the equation as a rank-k update to the error of the initial
    estimate.
mud.funs.parse_args(args)
    Parse command line parameters

    Parameters args ([str]) – command line parameters as list of strings

    Returns command line parameters namespace

    Return type argparse.Namespace

mud.funs.performEpoch(A, b, y, initial_mean, initial_cov, data_cov=None, idx=None)
mud.funs.run()
    Entry point for console_scripts
mud.funs.setup_logging(loglevel)
    Setup basic logging

    Parameters loglevel (int) – minimum loglevel for emitting messages

```

`mud.funs.updated_cov(X, init_cov=None, data_cov=None)`

We start with the posterior covariance from ridge regression. Our matrix $R = \text{init_cov}^{-1} - X.T @ \text{pred_cov}^{-1} @ X$ replaces the `init_cov` from the posterior covariance equation. Simplifying, this is given as the following, which is not used due to issues of numerical stability (a lot of inverse operations).

```
up_cov = (X.T @ np.linalg.inv(data_cov) @ X + R )^(-1)
up_cov = np.linalg.inv( X.T@(np.linalg.inv(data_cov)
- inv_pred_cov)@X + np.linalg.inv(init_cov) )
```

We return the updated covariance using a form of it derived which applies Hua's identity in order to use Woodbury's identity.

```
>>> updated_cov(np.eye(2))
array([[1., 0.],
       [0., 1.]])
>>> updated_cov(np.eye(2)*2)
array([[0.25, 0. ],
       [0.  , 0.25]])
>>> updated_cov(np.eye(3)[: , :2]*2, data_cov=np.eye(3))
array([[0.25, 0. ],
       [0.  , 0.25]])
>>> updated_cov(np.eye(3)[: , :2]*2, init_cov=np.eye(2))
array([[0.25, 0. ],
       [0.  , 0.25]])
```

`mud.funs.wme(X, data, sd=None)`

mud.norm module

`mud.norm.full_functional(operator, inputs, data, initial_mean, initial_cov, observed_mean=0, observed_cov=1)`

`mud.norm.inner_product(X, mat)`

Inner-product induced vector norm implementation.

Returns square of norm defined by the inner product $(x, x)_C := x^T C^{-1} x$

Parameters

- **X** ((M, N) array_like) – Input array. N = number of samples, M = dimension
- **mat** ((M, M) array_like) – Positive-definite operator which induces the inner product

Returns **Z** – inner-product of each column in **X** with respect to **mat**

Return type (N, 1) ndarray

`mud.norm.norm_data(operator, inputs, data, observed_mean, observed_cov)`

`mud.norm.norm_input(inputs, initial_mean, initial_cov)`

`mud.norm.norm_predicted(operator, inputs, initial_mean, initial_cov)`

mud.plot module

`mud.plot.make_2d_normal_mesh(N=50, window=1)`

`mud.plot.make_2d_unit_mesh(N=50, window=1)`

`mud.plot.plotChain(mud_chain, ref_param, color='k', s=100)`

`mud.plot.plot_contours(A, ref_param, subset=None, color='k', ls=':', lw=1, fs=20, w=1, s=100, **kws)`

mud.util module

`mud.util.null_space(A, rcond=None)`

Construct an orthonormal basis for the null space of A using SVD

Method is slight modification of `scipy.linalg`

Parameters

- **A** ((M, N) *array_like*) – Input array
- **rcond** (*float*, *optional*) – Relative condition number. Singular values *s* smaller than `rcond * max(s)` are considered zero. Default: floating point `eps * max(M,N)`.

Returns **Z** – Orthonormal basis for the null space of A. *K* = dimension of effective null space, as determined by `rcond`

Return type (N, K) ndarray

Examples

One-dimensional null space:

```
>>> import numpy as np
>>> from mud.util import null_space
>>> A = np.array([[1, 1], [1, 1]])
>>> ns = null_space(A)
>>> ns * np.sign(ns[0,0]) # Remove the sign ambiguity of the vector
array([[ 0.70710678],
       [-0.70710678]])
```

Two-dimensional null space:

```
>>> B = np.random.rand(3, 5)
>>> Z = null_space(B)
>>> Z.shape
(5, 2)
>>> np.allclose(B.dot(Z), 0)
True
```

The basis vectors are orthonormal (up to rounding error):

```
>>> np.allclose(Z.T.dot(Z), np.eye(2))
True
```

`mud.util.std_from_equipment(tolerance=0.1, probability=0.95)`

Converts tolerance *tolerance* for precision of measurement equipment to a standard deviation, scaling so that (100`probability`) percent of measurements are within *tolerance*. A mean of zero is assumed. *erfinv* is imported from *scipy.special*

`mud.util.transform_linear_map(operator, data, std)`

Takes a linear map *operator* of size (len(*data*), dim_input) or (1, dim_input) for repeated observations, along with a vector *data* representing observations. It is assumed that *data* is formed with $M@truth + \sigma$ where $\sigma \sim N(0, std)$

This then transforms it to the MWE form expected by the DCI framework. It returns a matrix *A* of shape (1, dim_input) and np.float *b* and transforms it to the MWE form expected by the DCI framework.

```
>>> X = np.ones((10, 2))
>>> x = np.array([0.5, 0.5]).reshape(-1, 1)
>>> std = 1
>>> d = X @ x
>>> A, b = transform_linear_map(X, d, std)
>>> np.linalg.norm(A @ x + b)
0.0
>>> A, b = transform_linear_map(X, d, [std]*10)
>>> np.linalg.norm(A @ x + b)
0.0
>>> A, b = transform_linear_map(np.array([[1, 1]]), d, std)
>>> np.linalg.norm(A @ x + b)
0.0
>>> A, b = transform_linear_map(np.array([[1, 1]]), d, [std]*10)
Traceback (most recent call last):
...
ValueError: For repeated measurements, pass a float for std
```

`mud.util.transform_linear_setup(operator_list, data_list, std_list)`

Module contents

1.5 Changelog

1.5.1 Versions 0.0.x

- Setting up initial repository, configuring CI/CD
- Migration of code from CU-Denver-UQ/mud-paper repo
- Revisions of architecture, moving modules around
- Rapid iteration, not sticking to semantic versioning
- Possible breaking versions between patches (some functions moved to *mud-examples*)
- Defines basic functionality, classes, helpful functions

1.5.2 Version 0.25

- Updated packaging to comply with PEP 517/518 using *pyscaffold* `v4.0.2`
- Removes *pyerf* in favor of *erfinv* from *scipy.special* (available since v0.2)
- Renames *testing* to *dev* for optional dependency installation
- Adds *black* as a *dev* dependency
- Run *black* + *flake8* on whole project
- clean up *setup.cfg* file
- adds file for readthedocs

1.5.3 Version 0.1

- Basic functionality
- Testing for all modules
- Beginning of adherence to semantic versioning rules
- i.e., breaking changes in major revision, contract changes in minor, bugfixes/features in patch.

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